Teaching Activity: Modeling Exponential Growth

Introduction: Probably the most significant factor in causing the resource and environmental predicament that we find ourselves in is the phenomenon of *exponential growth.* Over the past few decades, population, nonrenewable resource consumption, food production, industrial output and pollution generation have all been increasing exponentially. To understand the resource and environmental crises, one must understand exponential growth. Hopefully, this activity will afford you that understanding. It is built around the population problem, but it could just as well be applied to automobile production and disposal, electrical generating plant construction and demolition, etc.

Objectives:

- To use dice to model exponential growth;
  To change the variables and attempt to slow down and stop exponential growth;
- To relate this problem to human population trends and other human behaviors;

Materials: Dice or colored wooden cubes, paper, pencil, **Student Activity Sheets**, graph paper, colored pencils;

Procedure:

1. In this lab students will use dice or colored wooden cubes to model population growth.
   - Each die or cube represents a person.
   - Each throw represents a year.
   - The directions are written assuming they are using dice. If they are using wooden cubes, make the substitutions shown in the chart.

Event	Dice	Cubes
Birth	**3 or 6**	**White, red**
Death	**1**	**Black**

2. A three (3) or a six (6) represents the birth of a child, so each time one of them comes up, students should add a die to the population.
   (NOTE: **They are modeling a situation where the birth rate is twice the death rate, so they have a population growth rate of about 17%.)**

Part A: Unrestricted Exponential Growth (UEG)

3. Students should put 10 ordinary dice into a container (Adam, Eve, Cain, Abel, Sally, Alice, Dick, Jane, Bob and Sue) and then shake the container and dump the contents out onto a hard, smooth floor.
   - Students should remove and count all the ones (1s) that appear. A one is analogous to a death.
   - Students should record the number of deaths on the chart for Part A.

- Students should count up all the threes (3s) and sixes (6s) that appear. Since they correspond to births, a die should be added for each of them.
- Students should record the required information on the chart.

4. Students should repeat the above procedure until the population exceeds 500 people. If fewer than 500 dice are available, they need to figure out a way to get a full set of data using the dice they have.

Part B: The Effect of Instituting a Limited Birth Control Program (LBC)

5. Looking at the information on their chart, students should return to the year when the population was almost 100 people.
   - Students should put that many dice into the container.
   - A limited birth control program (LBC) will now be initiated. This will be modeled by saying that a 3 represents a birth, as before, and so does every other 6. However, the remaining half of the sixes represent women that are on birth control (or are married to men on birth control) and so a birth has been prevented.
   - If an odd number of sixes comes up, round off in favor of a birth half of the time, and in favor of a prevented birth the other half of the time. Students should model this situation, from where they start, for twelve throws of the dice. They have essentially cut the population growth rate from 17% to 8%.

Part C: The Zero Population Growth Plan

6. Students should return to the year where your population was almost 100 people. And put that many dice into the container.
   - A large scale birth control program is now being introduced. This will be modeled by saying that all the sixes represent women using effective birth control techniques, or women married to men using effective birth control techniques.
   - A one (1) represent a death, a 3 represent a birth and a 6 represents a prevented birth.
   - Students should model this situation from where they start for 12 throws of the dice.

Part D: World Population Trends

The following is a list of estimates of world population, 1650 through 1992. Figures such as these are compiled by the United Nations and are published in most almanacs.

1650 = .47 Billion	1930 = 2.1 Billion	1970 = 3.6 Billion
1750 = .69 Billion	1940 = 2.3 Billion	1980 = 4.4 Billion

1850 = 1.1 Billion	1950 = 2.5 Billion	1985 = 4.8 Billion
1900 = 1.6 Billion	1960 = 3.0 Billion	1992 = 5.5 Billion

7. Students should plot a graph of world population vs. time (1650-1992)

Part E: The Nature of Exponential Growth

8. Students should then examine the following 4 facts about exponential growth and relate them to their graphs.

   a. Exponential growth occurs whenever the rate of growth is directly proportional to the quantity of the material present.

   b. A quantity exhibits exponential growth when it increases by a constant percentage of the whole in a constant time period (like a savings account).

   c. Exponential growth is deceptive, because it starts out very slowly, but is capable of generating immense numbers very quickly.

   d. It is useful to think of exponential growth in terms of ***doubling time.***

$$\textbf{Doubling Time (in years)} = \frac{70}{\%\ \text{annual growth rate}}$$

Recent Growth Rates (1990):

Quantity	Annual Growth Rate	Doubling Time (in years)
World Human Population	1.7%	42
Nigeria's Population	2.9%	24
World Industrial Output	3.3%	21
Natural Gas Consumption	3.5%	20
Fertilizer Consumption	4.7%	15

GRAPHS:

Part A: Unrestricted Exponential Growth (UEG)

Students should use their data to plot graphs of :

- Population vs. Time (yr.)
- Population Growth Rate vs. Time (yr.)
- Population Growth Rate vs. Population

Part B: The Effect of Instituting a Limited Birth Control Program (LBC)

Students should use their data to plot a graph of:

- Population vs. Time

Part C: The Zero Population Growth Plan (ZPG)

Students should use their data to plot a graph of:

- Population vs. Time

Part D: World Population Trends

Students should use their to plot a graph of:

- World population vs. time (1650-1992)

Part E: LAB Write Up

- The **LAB Write Up** consists mainly of the 5 sets of data tables and graphs, and the conclusion students write for each one.

Student Activity Sheet : Modeling Exponential Growth

Introduction: Probably the most significant factor in causing the resource and environmental predicament that we find ourselves in is the phenomenon of *exponential growth.* Over the past few decades, population, nonrenewable resource consumption, food production, industrial output and pollution generation have all been increasing exponentially. To understand the resource and environmental crises, one must understand exponential growth. Hopefully, this activity will afford you that understanding. It is built around the population problem, but it could just as well be applied to automobile production and disposal, electrical generating plant construction and demolition, etc.

Objectives:

- To use dice to model exponential growth;
  To change the variables and attempt to slow down and stop exponential growth;
- To relate this problem to human population trends and other human behaviors;

Procedure:

1. In this lab you will use dice or colored wooden cubes to model population growth.
   - Each die or cube represents a person.
   - Each throw represents a year.
   - The directions are written assuming you are using dice. If you are using wooden cubes, make the substitutions shown in the chart.

Event	Dice	Cubes
Birth	**3 or 6**	**White, red**
Death	**1**	**Black**

2. A three (3) or a six (6) represents the birth of a child, so each time one of them comes up, add a die to the population.
   (NOTE: **You are modeling a situation where the birth rate is twice the death rate, so you have a population growth rate of about 17%.)**

Part A: Unrestricted Exponential Growth (UEG)

3. Put 10 ordinary dice into a container (Adam, Eve, Cain, Abel, Sally, Alice, Dick, Jane, Bob and Sue) and then shake the container and dump the contents out onto a hard, smooth floor.
   - Remove and count all the ones (1s) that appear. A one is analogous to a death.
   - Record the number of deaths on the chart for Part A.
   - Count up all the threes (3s) and sixes (6s) that appear. Since they correspond to births, a die should be added for each of them.
   - Record the required information on the chart.

4. Repeat the above procedure until the population exceeds 500 people. If fewer than 500 dice are available, figure out a way to get a full set of data using the dice you have.

Part B: The Effect of Instituting a Limited Birth Control Program (LBC)

5. Looking at the information on their chart, return to the year when the population was almost 100 people.
   - Put that many dice into the container.
   - A limited birth control program (LBC) will now be initiated. This will be modeled by saying that a 3 represents a birth, as before, and so does every other 6. However, the remaining half of the sixes represent women that are on birth control (or are married to men on birth control) and so a birth has been prevented.
   - If an odd number of sixes comes up, round off in favor of a birth half of the time, and in favor of a prevented birth the other half of the time. Model this situation, from where you start, for twelve throws of the dice. You have essentially cut the population growth rate from 17% to 8%.

Part C: The Zero Population Growth Plan

6. Return to the year where your population was almost 100 people. Put that many dice into the container.
   - A large scale birth control program is now being introduced. This will be modeled by saying that all the sixes represent women using effective birth control techniques, or women married to men using effective birth control techniques.
   - A one (1) represent a death, a 3 represent a birth and a 6 represents a prevented birth.
   - Model this situation from where you start for 12 throws of the dice.

Part D: World Population Trends

The following is a list of estimates of world population, 1650 through 1992. Figures such as these are compiled by the United Nations and are published in most almanacs.

1650 = .47 Billion	1930 = 2.1 Billion	1970 = 3.6 Billion
1750 = .69 Billion	1940 = 2.3 Billion	1980 = 4.4 Billion
1850 = 1.1 Billion	1950 = 2.5 Billion	1985 = 4.8 Billion
1900 = 1.6 Billion	1960 = 3.0 Billion	1992 = 5.5 Billion

7. Plot a graph of world population vs. time (1650-1992)

Part E: The Nature of Exponential Growth

8. Examine the following 4 facts about exponential growth and relate them to your graphs.

   a. Exponential growth occurs whenever the rate of growth is directly proportional to the quantity of the material present.

   b. A quantity exhibits exponential growth when it increases by a constant percentage of the whole in a constant time period (like a savings account).

   c. Exponential growth is deceptive, because it starts out very slowly, but is capable of generating immense numbers very quickly.

   d. It is useful to think of exponential growth in terms of ***doubling time.***

$$\text{Doubling Time (in years)} = \frac{70}{\%\ \text{annual growth rate}}$$

Recent Growth Rates (1990):

Quantity	Annual Growth Rate	Doubling Time (in years)
World Human Population	1.7%	42
Nigeria's Population	2.9%	24
World Industrial Output	3.3%	21
Natural Gas Consumption	3.5%	20
Fertilizer Consumption	4.7%	15

GRAPHS:

Part A: Unrestricted Exponential Growth (UEG)

Use your data to plot graphs of :

- Population vs. Time (yrs)
- Population Growth Rate vs. Time (years)
- Population Growth Rate vs. Population

Part B: The Effect of Instituting a Limited Birth Control Program (LBC)

Use your data to plot a graph of:

- Population vs. Time

Part C: The Zero Population Growth Plan (ZPG)

Use your data to plot a graph of:

- Population vs. Time

Part D: World Population Trends

Use the **Data Table** to plot a graph of:

- World population vs. time (1650-1992)

Part E: LAB Write Up

- The **LAB Write Up** consists mainly of the 5 sets of data tables and graphs, and the conclusion you write for each one.

Data: Part A: Unrestricted Exponential Growth (UEG)

Throw No. (Year)	Number of Births (N_b)	Number of Deaths (N_d)	Number of Dice (population)	Population Growth Rate* (Change per Year) Change per Year = $N_b - N_d$
0	-	-	10	-----
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
21				
22				
23				
24				
25				
26				
27				
28				
29				
30				

* This column is the population change per throw.

DATA: Part B: The Effect of Instituting a Limited Birth Control Program (LBC)

Throw No. (Year)	Number of Births (N_b)	Number of Deaths (N_d)	Number of Dice (population)	Population Growth Rate* (Change per Year) Change per Year = $N_b - N_d$)

*This change is the population change per throw.

Data: Part C: The Zero Population Growth Plan (ZPG)

Throw No. (Year)	Number of Births (N_b)	Number of Deaths (N_d)	Number of Dice (population)	Population Growth Rate* (Change per Year) Change per Year = $N_b - N_d$

* This column is the population change per throw.